

Entropic gravity, minimum temperature, and modified Newtonian dynamics

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Verlinde's heuristic argument for the interpretation of the standard Newtonian gravitational force as an entropic force is generalized by the introduction of a minimum temperature (or maximum wave length) for the microscopic degrees of freedom on the holographic screen. With the simplest possible setup, the resulting gravitational acceleration felt by a test mass m from a point mass M at a distance R is found to be of the form of the modified Newtonian dynamics (MOND) as suggested by Milgrom. The corresponding MOND-type acceleration constant is proportional to the minimum temperature, which can be interpreted as the Unruh temperature of an emerging de-Sitter space. This provides a possible explanation of the connection between local MOND-type two-body systems and cosmology.

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1. Introduction

In this Letter, we start from Verlinde’s heuristic argument¹ for the standard Newtonian acceleration on a test mass m from an effective point mass M at an effective distance R , the norm of the acceleration three-vector being given by GM/R^2 . In his approach, classical gravity arises as an entropic force, hence the name “entropic gravity.” Here, we will use a particular formulation^{2,3} of Verlinde’s argument, which relies only on the Unruh temperature and holography.

The new ingredient is the introduction of a minimum temperature $T_{\min} > 0$ for the fundamental microscopic degrees of freedom on the two-dimensional holographic screen. The goal of this Letter is to explore the consequences of having this minimum temperature. Interestingly, we will find that the simplest possible functional behavior is precisely of the type of Milgrom’s modified Newtonian dynamics (MOND) applied to nonrelativistic classical gravity.^{4,5,6}

It should be mentioned, right from the start, that the key equations of this article have appeared, in more or less the same form, in the previous literature. This article is primarily about concepts and logic. For this reason, the fundamental physical constants, \hbar , c , and k_B are occasionally displayed, even though typically we use units with $\hbar = c = k_B = 1$.

2. Setup

The N microscopic degrees of freedom on the spherical screen $\Sigma_{N,T,T_{\min}}$ are assumed to be in thermal equilibrium with a temperature

$$T = T_{\min} + \Delta T, \quad (1a)$$

$$T_{\min} > 0, \quad \Delta T \geq 0. \quad (1b)$$

An alternative description uses a maximum wavelength λ_{\max} for the thermal excitations (quasiparticles) of the microscopic degrees of freedom on the holographic screen. This (reduced) wavelength can be defined as follows:

$$c/\lambda_{\max} \equiv \frac{1}{2} k_B T_{\min} / \hbar. \quad (2)$$

Furthermore, the setup requires the following behavior for the macroscopic variables corresponding to the effective mass M and the area $A_{\Sigma} \equiv 4\pi R^2$:

$$M \propto N \Delta T, \quad A_{\Sigma} \propto N, \quad (3)$$

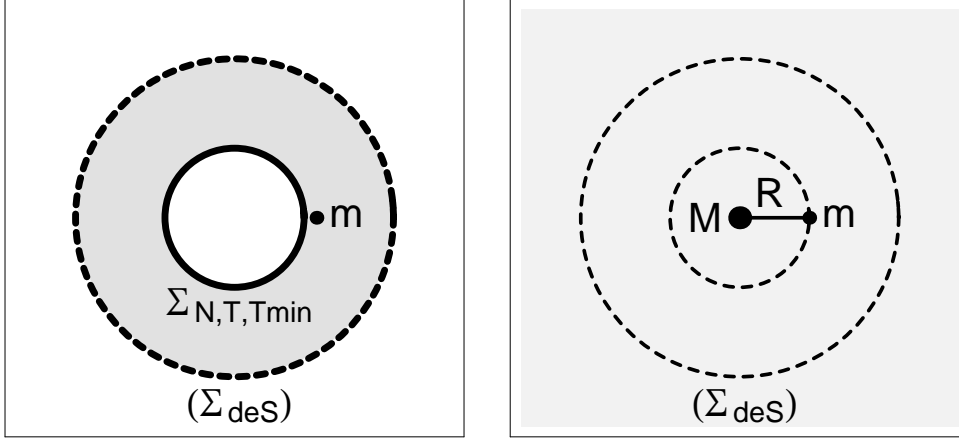


Fig. 1. Left panel: test mass m at rest in the emerged space (dark shading), just outside the spherical holographic screen $\Sigma_{N,T,T_{\min}}$ (full heavy curve). The fundamental microscopic degrees of freedom of the screen $\Sigma_{N,T,T_{\min}}$ have a minimum temperature T_{\min} and a corresponding event horizon (dashed heavy curve), which can possibly be identified as the de-Sitter horizon Σ_{deS} . Right panel: gravitational attraction experienced by the test mass m , as coming from an effective point mass $M \propto N(T - T_{\min})$ at an effective distance $R \propto N^{1/2}$ in a standard spacetime (light shading), possibly de-Sitter spacetime.

as will be discussed in Sec. 4.

The physical picture, now, is as follows. Having $T_{\min} > 0$ for the microscopic degrees of freedom of a given (inner) holographic screen $\Sigma_{N,T,T_{\min}}$ corresponds to having a nonzero entropy $S_{\min} > 0$. Such a nonzero entropy can be *interpreted* as being due to missing information⁷ from the presence of an event horizon for the degrees of freedom on the inner screen (“observers” in the usual terminology). From the holographic point of view,¹ having a maximum wavelength λ_{\max} for the microscopic degrees of freedom on the screen is certainly consistent with obtaining a finite length scale in the emerged space.^a

For the physics near the inner screen, it is important to understand that the event horizon is a derived effect and that what really matters is the heat-bath-type

^aThe event horizon can perhaps also be interpreted as an (outer) holographic screen Σ_{out} . It appears that the correct description is then that *each* holographic screen, $\Sigma_{N,T,T_{\min}}$ or Σ_{out} , has its *own* emerged space (a similar point has been made by Penrose⁸ in an entirely different context). Still, in order to describe the behavior of the test mass m near the inner screen, it may turn out to be useful to work in some type of “average space” between the two surfaces.

temperature T_{\min} of the microscopic degrees of freedom on the holographic screen; see the left panel of Fig. 1. The extra energy from an additional temperature ΔT of the degrees of freedom on the inner screen is responsible for a net attraction on a stationary test mass m just outside the screen (see Sec. 4 for details). According to Verlinde,¹ the resulting gravitational force \mathbf{F}_{grav} on a test mass m can be *interpreted* as coming from an effective point mass M at an effective distance R in an effective geodesically-complete spacetime; see the right panel of Fig. 1. It needs to be emphasized that the right panel of Fig. 1 is now considered to give only an approximate and derived description of the “physical reality,” whereas the left panel is taken to give a more accurate and more fundamental description.

3. De-Sitter realization

In the previous section, we have argued that the existence of an intrinsic minimum temperature T_{\min} for the degrees of freedom of the inner screen corresponds to the presence of an effective event horizon for these degrees of freedom. Now, identify this effective event horizon with the event horizon Σ_{deS} in an emerged de-Sitter (deS) space, so that T_{\min} equals the corresponding Unruh temperature.^{9,10,11,12,13} With the Gibbons–Hawking result¹⁰ $T_{\text{deS}} = H_{\text{deS}}/(2\pi)$ for a spherical event horizon at $r = c/H_{\text{deS}}$ in a static de-Sitter metric, we then have

$$2\pi T_{\min} = H_{\text{deS}} \equiv H, \quad (4)$$

where H is a useful short-hand notation.^b

Next, examine a detector with uniform linear acceleration \mathbf{A} in de-Sitter space. The resulting Unruh-type temperature has been calculated in Ref. 12: $(2\pi T)^2 = |\mathbf{A}|^2 + H^2$. Inverting this result, in the spirit of Ref. 1, and using (1a) and (4) gives

$$|\mathbf{A}| = \sqrt{(2\pi T)^2 - H^2} = 2\pi \Delta T \sqrt{1 + 2T_{\min}/\Delta T}. \quad (5)$$

The first equality in (5) can be understood as the correction to the acceleration associated with a local temperature T if Minkowski spacetime is replaced by de-Sitter spacetime (which has an event horizon even if the acceleration of the detector vanishes).

^bAn early paper¹⁴ on entropic gravity in a cosmological context also discusses a minimum temperature, but the setup of that paper is different from the one presented here.

From (5), a quadratic equation in ΔT is obtained, which has the following positive root:

$$2\pi \Delta T = \sqrt{|\mathbf{A}|^2 + (2\pi T_{\min})^2} - 2\pi T_{\min} . \quad (6)$$

For our purpose, de-Sitter space is only an auxiliary ingredient and we continue to work with the expression (6), solely 2 defined in terms of T_{\min} from the holographic screen.^c

Still, de-Sitter space is special, because the Unruh-type temperature T for a uniform linear acceleration of the detector is invariant under local Lorentz transformations of the detector motion.¹² The surprising role of special relativity in the Verlinde-type ‘derivation’ of standard Newtonian gravity has already been noted in Ref. 2. Apparently, the importance of local Lorentz invariance also holds for the ‘derivation’ of modified Newtonian gravity (see Sec. 4).

It may be that, for the case of a holographic screen with minimum temperature T_{\min} , the demand of local Lorentz invariance uniquely selects a de-Sitter space with a Hubble constant given by (4). But, for now, we simply *assume* de-Sitter space to be relevant or, at least, to provide a good approximation for the physics investigated.

4. Heuristic argument

At last, we are ready to calculate the gravitational attraction experienced by a stationary test mass m just outside the holographic screen $\Sigma_{N, T, T_{\min}}$ as shown in the left panel of Fig. 1. The procedure is simple: reverse (6) and use, starting from $2\pi \Delta T$, the Verlinde-type argument as given in Eq. (4) of Ref. 2. The norm of the inward radial acceleration \mathbf{A} of the test mass m generated by the screen quantities N , T , and T_{\min} is then found to be given by the following expression:

$$|\mathbf{A}| \hat{\mu} \left(\frac{|\mathbf{A}|}{4\pi c k_B T_{\min} / \hbar} \right) = 2\pi c k_B \Delta T / \hbar = GM/R^2 , \quad (7a)$$

^cObserve that the right-hand side of (6) is the simplest possible function of $|\mathbf{A}|$ which reduces to $|\mathbf{A}|$ for $T_{\min} = 0$, drops to 0 for $T_{\min} \rightarrow \infty$, and involves $|\mathbf{A}|$ only in the combination $|\mathbf{A}|^2 + (2\pi T_{\min})^2$.

with fundamental constants \hbar , c , and k_B restored and with definitions

$$\widehat{\mu}(x) \equiv \sqrt{1 + 1/(2x)^2} - 1/(2x), \quad (7b)$$

$$G \equiv f c^3 l^2 / \hbar, \quad (7c)$$

$$M \equiv \frac{1}{2} N k_B \Delta T / c^2, \quad (7d)$$

$$A_\Sigma \equiv f N l^2, \quad (7e)$$

$$R^2 \equiv \frac{1}{4\pi} A_\Sigma. \quad (7f)$$

Strictly speaking, the last step of ‘derivation’ (7a) is trivial, as it involves only mathematical definitions, *viz.* Eqs. (7c)–(7f). The real issue is, of course, to establish the corresponding physical picture. We start with six technical comments and, then, follow-up with a few general remarks. In a first reading, it is possible to skip these clarifications and to proceed directly to Sec. 5.

First, the quantity l^2 entering (7c) may (or may not) correspond to a new fundamental constant of nature, the quantum of area.^{2,3} The quantity f in (7c) is then an appropriate numerical factor appearing from the calculation of G (for $f = 1$, the length l equals the standard Planck length scale).

Second, macroscopic quantities in (7c)–(7f) are denoted by upper-case letters and fundamental constants by lower-case letters. More specifically, G , M , and R^2 are *effective* macroscopic quantities, derived from the fundamental quantities N and ΔT describing the microscopic degrees of freedom on the holographic screen.

Third, the behavior $N \propto A_\Sigma$ from (7e) corresponds to holography; see Ref. 1 for further discussion and references. The crucial assumption, here, is that N is a purely geometric quantity, that is, N is dependent on the area but not on the temperature (N is, for example, not proportional to the combination $A_\Sigma \Delta T / T$).

Fourth, given the number N of degrees of freedom on the screen, the extra energy $\frac{1}{2} k_B \Delta T$ per degree of freedom provides for an acceleration of the test mass m , which is absent in the perfect (empty, matter-free) de-Sitter space with $T = T_{\min}$ on the screen. In this way, it makes sense that the effective Newtonian mass M is defined to be proportional to N and ΔT , as shown by (7d). In fact, it is possible to imagine that the holographic screen consists of a gas of nonrelativistic “atoms of two-dimensional space.” The velocities of these identical atoms, $\{\mathbf{u}_n = \mathbf{v}_n + \mathbf{w}_n \mid n = 1, \dots, N/2\}$,

are assumed to be built from two sets of independent random velocities, $\{\mathbf{v}_n\}$ and $\{\mathbf{w}_n\}$, which give rise to T_{\min} and ΔT , respectively. The kinetic energy of the second set of random velocities, $\{\mathbf{w}_n\}$, then corresponds to the effective Newtonian mass M . Note that the corresponding gravitational force (7a) is not quite a standard entropic force (having $|\mathbf{F}| \propto T$) but a modified entropic force with a shifted temperature scale (having $|\mathbf{F}| \propto \Delta T \equiv T - T_{\min}$).

Fifth, it is possible to generalize the argument used in Eqs. (7a)–(7f) by allowing for modifications of the energy equipartition law of the microscopic degrees of freedom,³ but this is not necessary for the present discussion.

Sixth, an alternative ‘derivation’ of (7a) which directly starts from Verlinde’s entropic-force formula is given in the Appendix.

We now present the promised general remarks, intended to further clarify the physical picture (see Ref. 1 for additional details). These remarks are primarily concerned with the emergent space from the holographic screen and are highly speculative, because the fundamental theory is unknown (the ultimate goal is, of course, to learn something about this fundamental theory, a first clue perhaps having been found in Ref. 3).

By increasing or reducing the number N of degrees of freedom on the holographic screen the effective distance R between the masses M and m grows or shrinks, according to (7e)–(7f). In fact, reducing N corresponds to a coarse-graining of the degrees of freedom (similar to Kadanoff’s block-spin transformation in lattice models) and the resulting information (new coupling constants in the effective theory coming from the block-spinning) corresponds to an increased range of the orthogonal space coordinates, consistent with the picture of a shrinking surface at the inner boundary of the emerged space in the left panel of Fig. 1. For the present setup, the maximally coarse-grained surface is the Schwarzschild horizon.¹

Increasing N , while keeping M fixed, moves the screen out towards the de-Sitter horizon and the screen temperature T approaches T_{\min} from above, according to (7d). However, as discussed in Footnote a, the naive description in terms of a single emerged space can be expected to become invalid as the inner screen approaches the outer one.

5. Discussion

The first equality in (7a) already appears in a prescient paper by Milgrom,⁵ but the heuristic ‘derivation’ of the second equality is new and really makes for MOND applied to nonrelativistic classical gravity.⁴ The crucial extra input compared to Ref. 5 is the combination (7d) and (7e), see also the third and fourth technical comments in the previous section.

From the heuristic argument of the previous section or the one of the Appendix, the gravitational attraction of a stationary test mass m to a point mass M at a distance R (right panel of Fig. 1) is thus found to give the following inward acceleration \mathbf{A} of the test mass m :

$$\mathbf{A} \hat{\mu}(|\mathbf{A}|/A_0) = -(GM/R^2) \hat{\mathbf{n}}, \quad (8a)$$

with $\hat{\mathbf{n}}$ a unit vector pointing from M to m , the explicit function $\hat{\mu}(x)$ from (7b), having $\hat{\mu}(x) \rightarrow 1$ for $x \rightarrow \infty$ and $\hat{\mu}(x) \rightarrow x$ for $x \rightarrow 0$, and the acceleration constant

$$A_0 = 4\pi c k_B T_{\min}/\hbar = 8\pi c^2/\lambda_{\max}, \quad (8b)$$

in terms of the maximum wavelength defined by (2). As explained in Sec. 3, an effective de-Sitter space has been assumed to be relevant for the type of holographic screen considered and the corresponding horizon distance is given by

$$c/H_{\text{deS}} = \hbar c/(2\pi k_B T_{\min}) = 1/(4\pi) \lambda_{\max}. \quad (8c)$$

Eliminating T_{\min} (or λ_{\max}) from the last two equations gives

$$A_0 = 2 c H_{\text{deS}}, \quad (9)$$

which will be discussed later.

Note that (8a) can be expected to hold for linear motion (m moving towards or away from M) but not for circular motion (m orbiting M), relevant to the rotation curves of galaxies.⁴ The constant $a_0 \approx 1.2 \times 10^{-8} \text{ cm s}^{-2}$ obtained from the best available rotation-curve data⁶ can be expected to differ from our A_0 by a factor of order unity.¹¹ In addition, (9) is considered to hold for an exact de-Sitter space, but the present universe is not a perfect de-Sitter space, which will slightly change the temperature formula (5) and, thus, the resulting value of A_0 .¹³ Still, the order of magnitude of A_0 from (9) is quite reasonable, $A_0 \sim 10^{-7} \text{ cm s}^{-2}$, if H_{deS} is identified with $\sqrt{3/4} \approx 0.87$ times the measured Hubble constant $H_0 \approx 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [the

square root factor follows from the standard Friedmann equation of a spatially flat Universe with energy density ratio $\rho_{\text{vacuum}}/\rho_{\text{matter}} = 3$].

The possible relation of entropic gravity and MOND has been discussed in several recent papers; see, e.g., Refs. 15, 16, 17, 18, 19. Directly relevant to our discussion is the paper by Pikhitsa,¹⁹ of which we only became aware when writing up this Letter. Not surprisingly, his basic equations are the same as ours, but the precise claims and physical interpretation are different. For example, we do not claim to have obtained the MOND acceleration constant a_0 relevant for circular motion. And our direct physical interpretation of (7d) does not rely upon results from general relativity as appears to be the case for Eqs. (4)–(5) in Ref. 19. Still, our main physical conclusion is the same as Pikhitsa’s, namely, that MOND may be related to the existence of a minimum temperature. However, in the spirit of Verlinde’s approach,¹ we reverse cause and effect: a minimum temperature (maximum wave length) of the fundamental microscopic degrees of freedom responsible for classical gravity may produce a MOND-like behavior at sufficiently small accelerations of the test mass.

The question arises as to the nature of the holographic screen if the minimum temperature of its degrees of freedom is indeed nonzero. One possible explanation is that these fundamental microscopic degrees of freedom of the screen are in a long-lived metastable state. (Having such a metastable state may not be altogether unreasonable if the microscopic degrees of freedom have long-range interactions as has been argued to be the case in Ref. 3.) If the interpretation as a metastable state is correct, then there is, in principle, the possibility of a discontinuous reduction of the MOND-type acceleration constant (8b). In turn, this may lead to a discontinuous decay of the corresponding de-Sitter spacetime (it is not clear if there is any relation with the type of de-Sitter decay recently discussed by Polyakov²⁰).

Let us, finally, return to (9), which relates a characteristic, A_0 , of small-scale two-body dynamics (8a) to a cosmological quantity, H_{deS} . *A priori*, such a relation would be hard to understand. But this article suggests that both quantities (the “local” A_0 and the “global” H_{deS}) have a *common origin*. As shown by the left panel of Fig. 1, the suggestion is that a minimum temperature T_{min} (or maximum wave length λ_{max}) of the fundamental microscopic degrees of freedom on the holographic screen gives rise to both the MOND-type acceleration constant $A_0 \sim T_{\text{min}}$ and the

de-Sitter horizon distance $1/H_{\text{deS}} \sim 1/T_{\text{min}}$.

Granting the approximate equality both of A_0 and the inferred MOND acceleration constant a_0 for circular motion and of H_{deS} and the measured Hubble constant H_0 from the expanding Universe, there is then a logical connection between a_0 and H_0 , resulting in the relation $a_0 \sim c H_0$ (the approximate numerical coincidence of a_0 and $c H_0$ was already noted in Milgrom's original paper⁴). The logical connection between a_0 and $c H_0$ is indirect, as each of them traces back to the apparently more fundamental quantity T_{min} . Of course, all this only makes sense if the heuristic argument used here is physically relevant.

Appendix A. Alternative heuristic argument

In this appendix, the main result in (7a) is obtained by directly following Verlinde's original argument.¹ The starting point is the entropic-force formula ($\hbar = c = k_B = 1$)

$$\mathbf{F}_{\text{grav}} = \left[T \nabla S \right]_{\Sigma_0}, \quad (\text{A.1})$$

for a single spherical holographic screen Σ_0 with area $4\pi R^2 \propto N$ and effective mass $M_0 = \frac{1}{2} N T$, combined with the assumption

$$\nabla S = -2\pi m \hat{\mathbf{n}}_0, \quad (\text{A.2})$$

for unit normal $\hat{\mathbf{n}}_0$ of the screen Σ_0 directed towards the particle with mass m (the particle is separated from the screen by a distance of the order of its Compton wave length, \hbar/mc).

The basic idea, now, is to replace the original entropic-force formula (A.1) by

$$\mathbf{F}_{\text{grav}} = \sum_{n=1}^{2K} \left[T \nabla S \right]_{\Sigma_n}, \quad (\text{A.3})$$

where the sum includes three types of contributions and has an integer $K \gg 1$ to control the number of terms. The first type of contribution in (A.3) comes from the main spherical screen $\Sigma_{N,T,T_{\text{min}}} \equiv \Sigma_1$ discussed in Sec. 2. The second type of contribution comes from a plane screen Σ_2 with $T = T_{\text{min}}$, where Σ_2 is orthogonal to the $\hat{\mathbf{n}}_1$ normal from Σ_1 passing through the particle (specifically, $\hat{\mathbf{n}}_2 = -\hat{\mathbf{n}}_1$) and Σ_2 is positioned on the other side of the particle m compared to Σ_1 . The third type of contribution comes from many $(K - 1 \gg 1)$ pairs of parallel plane $T = T_{\text{min}}$ screens

having different random orientations ($\hat{\mathbf{n}}_n \times \hat{\mathbf{n}}_1 \neq 0$ for $n \geq 3$) and sandwiching the particle between them. For simplicity, the pure de-Sitter screens have been taken to be infinite planes, rather than spheres with very large radii ($c/H_{\text{deS}} \gg R$). The particle is thus surrounded by $2K - 1$ plane screens Σ_n (for $n = 2, \dots, 2K$) with temperature T_{min} and a single spherical screen Σ_1 with temperature $T \geq T_{\text{min}}$. The corresponding physical picture is effectively that of a particle m immersed in an anisotropic heat bath due to de-Sitter space and the localized energy density.

Using (A.2) and the de-Sitter-space Unruh temperatures,^{10,12} all matching contributions from pure de-Sitter screens cancel in the sum (A.3) and we are left with only two contributions (from the screens Σ_1 and Σ_2):

$$\mathbf{F}_{\text{grav}} = -m \left(\sqrt{|\mathbf{A}|^2 + (2\pi T_{\text{min}})^2} - 2\pi T_{\text{min}} \right) \hat{\mathbf{n}}_1. \quad (\text{A.4})$$

Simply taking over Verlinde's ‘derivation’ (Sec. 3.2 of Ref. 1) of the standard Newtonian gravitational force $|\mathbf{F}_{\text{grav},0}| = m G M_0 / R^2$ for the $T_{\text{min}} = 0$ case (corresponding to Minkowski spacetime) gives for the norm of (A.4) in reversed order:

$$m \left(\sqrt{|\mathbf{A}|^2 + (2\pi T_{\text{min}})^2} - 2\pi T_{\text{min}} \right) = m (G M_0 / R^2) \left(1 + \mathcal{O}(T_{\text{min}}/T) \right), \quad (\text{A.5})$$

which, to leading order in T_{min}/T , reproduces the behavior of (7a). Note that the mass M_0 times the last factor 2 in brackets of (A.5) corresponds to the mass M defined by (7d).

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